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11

AN EXPERIMENT ON THE SIMULATION OF CULTURE CHANGES

(WITH KENNETH L. COOKE)

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The problem of the long-term analysis of culture change for specific cultures has long been discussed by philosophers of history such as Spengler (1918) and Toynbee (1947), although the resulting insights have usually been expressed as a series of rather vague generalisations.

The development of systems theory has for some years suggested the possibility that the simulation of the historical trajectories of culture systems might be undertaken using a quantitative approach, although the complexity of any real culture system is clearly very great.

This chapter describes a preliminary and largely unsuccessful attempt to apply this approach. We believe, however, that the attempt is an interesting one, illustrating as it does a simple interaction model, and the various practical decisions which have to be made in applying it. The outcome was not a successful, even if crude and oversimplified, representation of change of a prehistoric culture. Instead we were able to recognise, and then analyse, the manner in which the form of the model, whatever the nature of the actual data used as input, came to determine the 'predicted' behaviour of the system. Comparison with the well-known 'world model' used for the study *The Limits to Growth* (Meadows *et al.* 1972) led in an illuminating way to an analogous conclusion: that at this stage criticism of its assumptions about the present state of the world (i.e., about input data) are largely irrelevant. The nature of the output of that model, like our own, is best appreciated after critical examination of its structure, and of the range of outputs of which it is in fact capable.

The object of this chapter is therefore to describe an experiment in systems modelling during which we gained some valuable experi-

ence of the real problems involved. The work arose out of discussions at the Center for the Study of Democratic Institutions, Santa Barbara, California, in which a number of approaches to the use of mathematical modelling, systems theory, and simulation in tracing culture trajectories were discussed by various participants including Wilkinson and co-workers (1973), Rosen (1973), and the authors of the present chapter (cf. Cooke 1973).

The Interaction Model

The problem for which this model was formulated is the explanation of the emergence of complex societies in the Aegean in the third and second millennia BC (Renfrew 1972). The approach is one of very general application, however, and with just a few modifications can be applied to the early development of any complex society.

The fundamental theoretical proposition sustaining the model – with which few will disagree – is that the changes and developments in the cultures in question (in this case the Aegean) were brought about by complex interactions of components in the society. As it stands this asserts no more than the rejection of very simple migrationist or diffusionist views, and naturally leaves room for the assessment of the effects within the society of contacts with other areas.

In the language of systems theory, the society is a system, its components being the members of the society, all the artefacts they made or used (including nonmaterial ones), and all objects in nature with which they came in contact. In this view, there is an essential coherence or conservatism of cultures due to negative feedback. That is, the system acts to counter disturbances. Innovations in society, on the other hand, are due to 'deviation-amplifying mutual causal processes', that is, positive feedback.

In order to use these concepts, it is necessary to decompose the society or system as a whole into a number of subsystems, and then to analyse in detail the interactions among these subsystems. A large part of the work cited (Renfrew 1972) is devoted to this identification of subsystems and their interactions. The subsystems are briefly defined as follows (see chapter 9):

1. Subsistence subsystem. The interactions that define this system are actions relating to the exploitation of food resources. Man, the food resources, and the food units themselves are components of the subsystem.

2. Metallurgical technology.

3. Craft technology. The metallurgical and craft subsystems are

defined by the activities of man which result in the production of material artefacts. The components are the men, the material resources, and the finished artefacts.

4. The social subsystem. This is a system of behaviour patterns, where the defining activities are those which take place between men.

5. The projective or symbolic subsystem. Here we are speaking of all those activities, notably religion, art, language, and science, by which man expresses his knowledge, feelings, or beliefs about his relationship with the world.

6. The external trade and communications subsystem. This is defined by all those activities by which information or material goods are transferred across the boundary of the system between human settlements or over considerable distances.

Population does not strictly rank as a subsystem within the framework described here, not representing a class of activity, but a parameter relevant to the description of the system.

This brief description does not begin to describe adequately the analysis of the system into its component subsystems. Nor is it claimed that the subsystems defined represent the only or the best such breakdown. For most cultures, for instance, it would not be desirable to distinguish the metallurgical and craft technology subsystems. What is essential to this approach, however, is some such analysis into subsystems, and an evaluation of the extent to which they interact, so that growth and development in one favours or inhibits development in another to an extent which can be made explicit. Such interactions have been considered in some detail for the Aegean (summarised in chapter 10, above). The matrix of interactions (table 1, p.293) is thus of particular importance. The projection and quantification of such a matrix is a necessary step in an analysis of this kind (cf. Plog 1974, tables 6.1 and 13.1).

Any causal chain, using this framework of analysis, can be illustrated diagrammatically as a number of feedback loops, where the subsystems which interact strongly are seen as linked (with a sign to indicate whether feedback is positive or negative). The values (or functions) in the matrix indicate the strength (or the variation) in the feedback for each pair of subsystems. Likewise any matrix of interactions can be represented diagrammatically: When weak or zero linkages are omitted from the diagram the complex web of interactions is more easily seen as a series of feedback loops.

The Simulation: Objectives and Technique

In all simulations, one wishes to gain insight into the structure of the system by tracing out its performance under a variety of suppositions concerning that structure and under a variety of conditions. Thus, in these experiments, we are interested in the evolution of culture when the subsystems interact as in table 1 (p.293), as well as when alternative interaction matrices are assumed.

Methods for bringing this system within the compass of concrete mathematics technique have been suggested by Cooke (1973) and Rosen (1972, 1973). Here we shall report on experiments with a version of Rosen's method. As in most systems analysis, we must first identify the variables. For us, the variables are in six subsystems listed earlier plus population. Of course, it would be possible to break each subsystem down into smaller parts, for purposes of greater detail, but in this formative stage it has seemed preferable to use a simple, highly aggregated model.

Some comment on 'population' as a variable is required. We mean to measure here the actual size of the population and to ignore all other demographic factors. It could well be argued that population density, age distribution, genetic factors, and so on, are of importance. However, it is convenient for the present discussion to restrict our application to Crete during the Early Bronze Age. Since the land area is thus fixed, there is no need to distinguish between population and population density. Moreover, there is no persuasive evidence of significant racial or genetic change during this period. Also, the available data could be interpreted to indicate an overall regularity of population growth, making it unnecessary to postulate important fluctuations in age distribution, and so on. Thus, population is the chosen variable. Incidentally, we make the usual convenient assumption in deterministic mathematical models that population can be treated as a real variable in the mathematical sense, rather than as an integer variable.

It is now necessary to decide what values the other variables shall be allowed to take. Although there is a possibility of using discrete or qualitative values, we have here elected to allow each variable to take real values.

Finding an appropriate real variable as an indicator for a concept is an important and difficult task. In our case, we seek a variable descriptive of the state of each of the subsystems. One of the ways in which this can be done is to regard each variable as an *index* of the level of sophistication of the subsystem it measures. We have

adopted this approach here. It should of course be noted that for the purposes of analysis the state of each subsystem is here regarded as adequately described by a single index. Thus we define x_i to be a numerical variable whose values represent a scale for this variable. In our case, we let $x_i(t)$ be the measure of the i th subsystem at time t , where the subsystems corresponding to $i = 1, 2, 3, 4, 5, 6$, are those indicated earlier, in the order given. We also let Cx_7 be the number of persons in the population, where C is a scaling constant to be chosen for convenience.

In archaeological terms the precise choice of the measure used to give a value for each of the variables is of course a crucial one, and in practice the behaviour of the system will be in part determined by the measure chosen. In each case several approaches are possible. The following formulations are set out simply to show that the problems, although of major theoretical significance to archaeology, do not prevent the selection of indices whose values can in principle be estimated from the archaeological record. For example, we may define the indices as follows:

x_1 (subsistence). The number of persons fed by the food-gathering-food-producing activities of one person.

x_2 (metallurgy). The number of metal artefacts produced within the territory of the society per head of population per year.

x_3 (craft). The range of specialist products in the society, measured in terms of artefact types.

x_4 (social). The number of well-defined roles distinguished in the society.

x_5 (projective). *Either* the number of abstract concepts in use in the society relating to measure (including units and numerals) *or* the number of man-hours per head per year spent in religious observances or in facilitating them (e.g., by building temples).

x_6 (external trade). Proportion of the GNP exported beyond the confines of the system.

Because of the mathematical formalism to be adopted here, it is necessary to *normalise* each variable so that its value lies between 0 and 1. This can be done in various ways. As an example, suppose that x_1 is defined as just suggested, and suppose that it is estimated that x_1 takes values between 1.05 and 1.8 for the class of cultures under study. Then if we let $\bar{x}_1 = x_1/3$, the values of $\bar{x}_1$ lie between 0.35 and 0.6. These values are certainly between 0 and 1, and values near 1 represent a better developed subsistence system than values near 0. The population variable is normalised by choice of the constant C . For example, we might estimate that the population of

Crete was 12,600 in the Middle Neolithic period. If we choose $C=630,000$, then $x_7=0.02$ at this time. Subsequent population increases correspond to increasing values of x_7 .

We shall henceforth assume that all variables have been normalised, and that measures have been selected for the variables in such a way that the expert in cultures is able to correlate these numerical values with his qualitative perceptions.

A second alternative interpretation of the variable has been suggested by Rosen (1973). Roughly speaking, $x_1(t)$ is regarded as a measure of the percentage of time spent by all members in the population on subsistence-related activities, $x_2(t)$ is a measure for metallurgy-related activities, and so on. (Consequently, the sum of $x_1 - x_6$ must equal 1 at all times.) This interpretation has not been adopted here, for reasons explained at the end of this section.

The next stage in the simulation is to adopt a procedure for changing the values of the variables. The key to this is the matrix of interactions, which is reproduced in table 1 (p.293).

The matrix is to be interpreted in the following way. Growth in a subsystem y in the left column is favoured by growth in a subsystem x in the top row through the mechanism indicated in the matrix. The key to the mechanisms A1, A2, . . . , G7 may be found in chapter 10. As suggested by Rosen, we replace this matrix by the matrix of *signatures*, as follows. Wherever no interaction is indicated, we place a 0. Wherever a favouring mechanism appears, we place +1. Wherever an inhibiting mechanism appears, we place -1. Thus, we obtain a matrix, which we name A , as follows:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Later, we shall discuss possible changes in the matrix A .

The procedure we have adopted for using the signature matrix has been suggested by Kane (1972) and Kane *et al.* (1972, 1973). First, a choice of the *initial state* of the system is made. That is, values are assigned to the variables at time $t=0$, corresponding to an estimate of the stage of development of the subsystems. Next, a sequence of subsequent states (i.e., values of the variables) is calculated by mathematical iteration. Consider a system with any

number m of variables. (In the application previously described, $m=7$.) If h denotes one unit of time, the values are determined from the equation

$$x_i(t+h) = x_i(t)^{p_i} \quad (i = 1, 2, \dots, m) \quad (1)$$

where t is given the successive values $t=0, h, 2h, \dots$, and where the p_i are exponents calculated as follows:

$$p_i = \frac{q_i}{r_i} \quad (2)$$

$$q_i = 1 + \frac{1}{2}h \sum_{k=1}^m (|a_{ik}| - a_{ik})x_k(t) \quad (3)$$

$$r_i = 1 + \frac{1}{2}h \sum_{k=1}^m (|a_{ik}| + a_{ik})x_k(t) \quad (4)$$

If we let

$$a_{ik}^- = \begin{cases} |a_{ik}| & \text{if } a_{ik} < 0 \\ 0 & \text{if } a_{ik} \geq 0 \end{cases} \quad (5)$$

$$a_{ik}^+ = \begin{cases} 0 & \text{if } a_{ik} \leq 0 \\ a_{ik} & \text{if } a_{ik} > 0 \end{cases} \quad (6)$$

then

$$q_i = 1 + h \sum_{k=1}^m a_{ik}^- x_k(t) \quad (7)$$

$$r_i = 1 + h \sum_{k=1}^m a_{ik}^+ x_k(t) \quad (8)$$

If A^+ denotes the matrix with entries a_{ik}^+ , A^- denotes the matrix with entries a_{ik}^- , and x denotes the vector with entries x_i , then we may write

$$q_i = 1 + h(A^-x)_i \quad r_i = 1 + h(A^+x)_i \quad (9)$$

where $(\)_i$ denotes the i th component of a vector.

From equations (1) we can generate values of the variables x_i at successive time points $t=0, h, 2h, \dots$. The equations are nonlinear, but have certain useful features. First, if $0 \leq x_i(t) \leq 1$ for all values of i , then q_i and r_i are both positive, as can be seen from (7) and (8) or (9). Consequently, p_i is positive and therefore $0 \leq x_i(t+h) \leq 1$.

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That is, the region $0 \leq x_i \leq 1$ ($i = 1, 2, \dots, m$) is invariant (variables automatically remain in the prescribed domain). Thus, the equations are suitable for use with variables scaled to lie between 0 and 1. We note also that if $x_i(t) = 0$ or 1 for some value of i , then $x_i(t+h) = 0$ or 1. The exponent p_i incorporates the time step h in order to adjust the time scale. Another important property of these equations is that positive feedbacks ($a_{ik} > 0$) increase the number r_i but do not contribute to q_i , and negative feedbacks ($a_{ik} < 0$) increase the number q_i but do not contribute to r_i . It follows that positive feedback tends to make p_i small and negative feedback tends to make p_i large. If $0 < x_i(t) < 1$, we see from (1) that $x_i(t+h) > x_i(t)$ when $p_i < 1$ and $x_i(t+h) < x_i(t)$ when $p_i > 1$. Thus, a preponderance of positive feedback will result in increasing values of x_i and a preponderance of negative feedback will result in decreasing values. More precisely, if

$$\sum_{k=1}^m a^+_{ik} x_k(t) > \sum_{k=1}^m a^-_{ik} x_k(t) \quad (10)$$

then $p_i < 1$ and $x_i(t+h) > x_i(t)$ [unless $x_i(t) = 0$ or 1]. Additional discussion of the mathematical properties of (1) is given later.

Underlying this procedure is the assumption that a complex interacting system can be adequately described by a set of binary interactions between pairs of variables – that is, that the influence of x_k on x_i is independent of the values of all variables other than x_k . This assumption is frequently made in modelling schemes based on the concept of feedback loops.

It will now be explained why it is not convenient to interpret $x_i(t)$ as a measure of the percentage of time spent by all members of the population on activities related to the i th subsystem. To be more precise, Rosen suggested that for each subsystem i , and for each person p in the population, we let $f_i(p)$ denote the proportion of time or degree of involvement of p with activity i ; thus $0 \leq f_i(p) \leq 1$ and $\sum_{i=1}^m f_i(p) = 1$. Then, summing for all the persons p in the population, we let

$$u_i = \sum_{p=1}^N f_i(p)$$

Finally, define $x_i = u_i/N$, where N is the total number of persons in the population, so that $0 \leq x_i \leq 1$. Now clearly we have

$$\sum_{i=1}^m x_i = \frac{1}{N} \sum_{i=1}^m u_i = \frac{1}{N} \sum_{i=1}^m \sum_{p=1}^N f_i(p) = \frac{1}{N} \sum_{p=1}^N \sum_{i=1}^m f_i(p) = \frac{1}{N} \sum_{p=1}^N 1 = 1$$

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That is, the normalised variables x_i must sum to 1. However, if we start with variables x_i at time $t=0$ which sum to 1 and apply equation (1), the new values x_i [i.e., $x_i(h)$] will not, in general, sum to 1. Thus, the use of equation (1) is incompatible with this interpretation of the variables x_i .

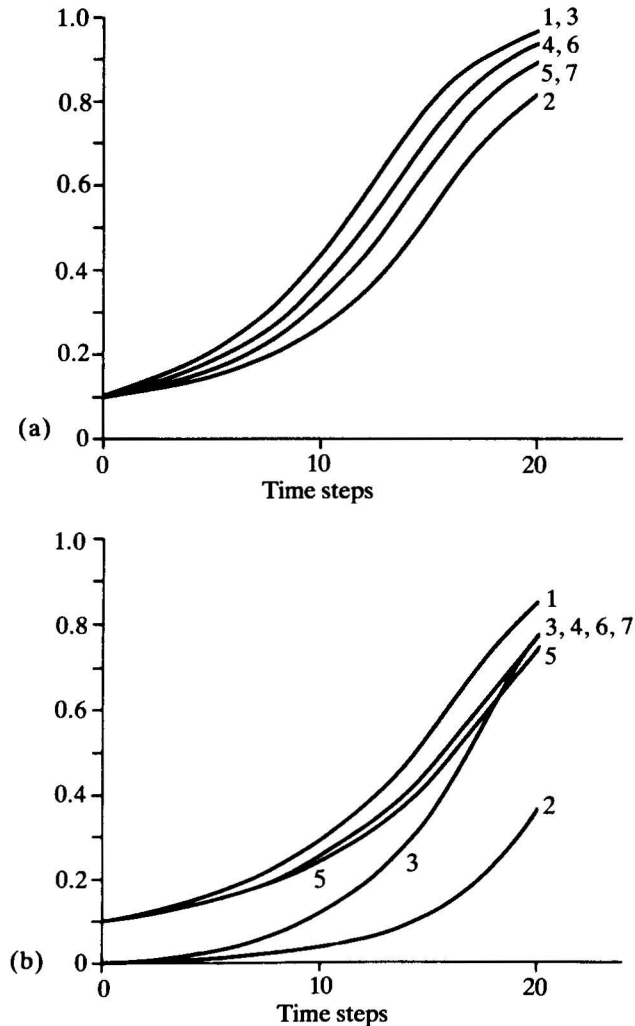


Figure 1. Growth pattern of the variables using signature matrix A. Initial values: (a) all at 0.1 (b) variables 2 and 3 at 0.01, remainder 0.1.

Initial Trial Runs

A number of trials were first made to test the behaviour of our system. Simulations were carried out on the computer at Claremont using a program in APL. Visual interactive facilities were used, allowing progressive experimentation with input parameters. All the figures have been re-drawn from Polaroid photographs of the screen display. Figures 1a and b show the growth pattern of the variables when the signature matrix A given in the preceding section is used, with two different choices of the initial conditions. In the first case, all variables are started with initial values of 0.1. In the second, variables 2 and 3 (metallurgy and craft technology) are started with values of 0.01. Figure 2 shows the growth when the initial values are all 0.1 but the signature matrix is changed by arbitrarily placing zeros in all entries in the fourth column to simulate the hypothesis that there is no positive feedback from the social subsystem to other subsystems.

Examination of these graphs shows that in all cases, the variables follow an S-shaped growth curve. That is, there is slow growth at first, then approximately exponential growth, and then levelling off toward the maximum possible level. Certain curves tend to lie above the others, notably the graph of x_i (subsistence). However, little can be inferred from this at this time, since no calibration of the numerical scales has been adopted.

An implicit assumption in the use of signature matrices of this kind is that the feedbacks are all of equal strength. Whatever the significance of the numerical values, whatever their scalings, this seems unlikely to be true. In fact, there is no reason to restrict the entries to being 1, 0, or -1 . The mathematical formulas allow any real numbers. By calling on panels of experts, one can assign different weights to different influences. The following matrix was suggested, as a guess at appropriate weightings of influences, by E. Elster and S. LeBlanc:

$$B = \begin{bmatrix} 0 & 0.50 & 0.50 & 1.00 & 0.25 & 0.75 & 0.75 \\ 0 & 0 & 0.75 & 2.00 & 0 & 1.00 & 0 \\ 0.50 & 0.75 & 0 & 0.50 & 0.50 & 0.25 & 0.25 \\ 0.25 & 0.75 & 0.50 & 0 & 0.10 & 0 & 0.50 \\ 0 & 0 & 0.25 & 0.75 & 0 & 0.25 & 0.50 \\ 0.25 & 1.00 & 0.75 & 0.35 & 0.10 & 0 & 0 \\ 0.75 & 0 & 0.1 & 0.85 & 0.10 & 0 & 0 \end{bmatrix}$$

The program was run with initial values 0.3, 0.01, 0.1, 0.1, 0.1, 0.2,

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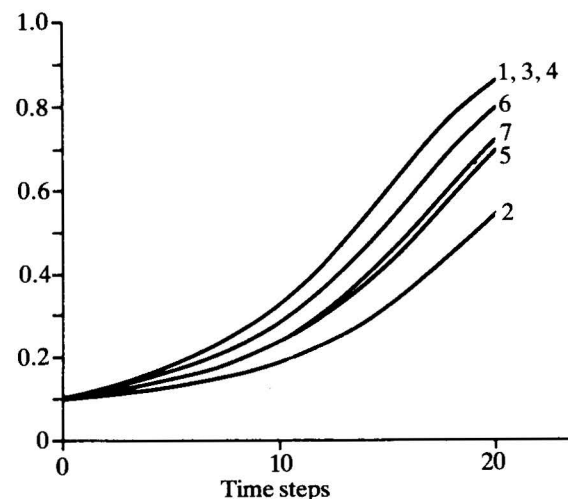


Figure 2. Growth pattern of the variables using matrix A with zero for all entries in the fourth column.

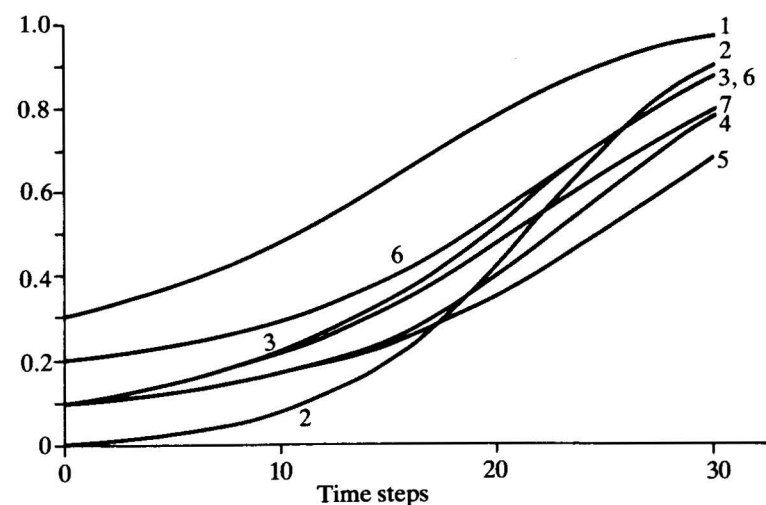


Figure 3. Growth pattern of variables using matrix B and initial values between 0 and 0.3.

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0.1, and the results are shown in figure 3. The chosen initial values reflect an assumption that subsistence and trade were at a higher level than the other subsystems at the beginning of the Early Bronze Age.

The choice of a weighting matrix B such as the one given here again raises questions about the scaling. For example, if we consider any row of the matrix, the sum of its entries in some sense indicates the total influence of all variables on the subsystem associated with that row. The row sums for the B matrix are as follows: subsistence 3.75, metallurgy 3.75, craft 2.75, social 2.1, projective 1.75, trade 2.45, population 1.8.

It will be noted from figure 3 that a large row sum tends to make the corresponding variable increase rapidly but, because of secondary effects, the relationship is not strict.

Each column in B represents the total effect of one subsystem on all others. For the matrix given here, these column sums are 1.75, 3, 2.85, 5.45, 1.05, 2.25, 2. This gives the impression that the influence of the social subsystem is being weighted far more heavily than the influence of any other subsystem.

A problem that arises in interpreting these simulations is the determination of the scale of the time variable. What is the period of time to be assigned to the arbitrary step h , the interval between successive states of the iteration?

In any real case some approximate estimate for the rate of change can be reached for most of the variables, and from this the length of the time step may be established. To take one example, estimates exist for the population of Crete at successive time periods. For instance, it has been suggested, although the figures are hypothetical, that the estimated population of Crete rose from 12,600 to 75,000 in about 1500 years. Therefore x_7 increased by a factor of 6 in 1500 years. Consulting figure 1a we see that x_7 increased from 0.1 to 0.6, that is, by a factor of 6, in about 14.4 time steps. Therefore we can assume that one time step represents about 1500/14.4, or 104 years. The same calculation applied to figure 1b results in a time step of $h = 86$ years, and figure 2 in 84 years. Of course it must be emphasised that this calculation depends on the assumption that the growth curve produced by the model represents the actual growth curve to within acceptable accuracy, and that the estimates in question make a suitable basis for calculation. Any other variable could be used in a similar manner.

As we have observed, all the results so far show steady growth of all subsystems. In fact, in all the experiments described so far, all

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entries in the matrix are positive. Therefore, (10) holds and all variables must increase at every time step. When all feedbacks are positive, we have a pure growth model. When all feedbacks are negative, we have a pure decay model. Moreover, in the pure growth model, since each variable is increasing, and yet cannot exceed 1, it must level off either at 1 or possibly at some value less than 1. In the following section, we show that only in exceptional cases can the variables approach equilibrium values less than 1. Consequently the expected behaviour is for all variables to follow an S-shaped growth levelling off at 1.

It is clear that no culture system behaves in such a way that all variables increase to some high notional value, and remain indefinitely at this high level. Indeed, as used up to this point, the model, although adequately simulating growth, fails to reflect adequately the property that is most characteristic of all successful culture systems: homeostasis.

A preliminary experiment was to insert some negative values for the indices in one subsystem. Here 'piracy' was imagined as operating negatively on a number of subsystems. (The projective system was omitted in this instance.) The Matrix C is as follows:

	Subs.	Met.	Cr.	Soc.	Tr.	Pop.	Pir.
Subs.	0	0.50	0.50	1.00	0.75	0.75	-2.00
Met.	0	0	0.75	1.00	1.00	0	-0.2
Cr.	0.50	0.75	0	0.50	0.25	0.25	0.3
Soc.	0.25	0.75	0.50	0	0	0.50	0.50
Tr.	0.25	1.00	0.75	0.35	0	0	-2.0
Pop.	0.75	0	0.10	0.85	0	0	-0.5
Pir.	0.5	0.75	0.75	0.50	1.00	0.50	0

All variables were given initial values of 0.3, and the results are shown in figure 4. The negative interactions in this case slow down growth of some variables but do not cause a collapse of the system.

Range of Behaviour of the Iterates

After a number of trial runs such as the foregoing, it became apparent that a more thorough examination of the properties of the system was required. What behaviour, other than steady growth and steady decay, is possible for the mathematical iteration given by equations (1)–(9)? In attempting to answer this question, we have proceeded in two ways. On the one hand, we have used theoretical

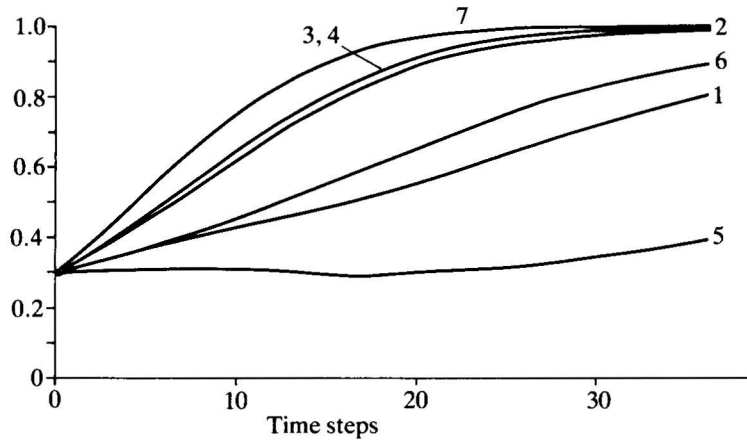


Figure 4. Growth pattern of the variables with some negative values for the indices (matrix C). All variables have initial value 0.3.

means to locate all possible equilibrium points of the iteration. On the other hand, we have used computer experiments to demonstrate that sustained periodic oscillations can occur.

We begin by describing our results concerning equilibrium states. We let x denote the column vector with components x_i , and write $p_i(x, h)$ instead of p_i to emphasise the dependence on x and h . Let

$$f_i(x, h) = x_i^{p_i(x, h)} = \begin{cases} \exp[p_i(x, h) \log x_i] & \text{if } x_i > 0 \\ 0 & \text{if } x_i = 0 \end{cases} \quad (11)$$

and let $f(x, h)$ be the vector with components $f_i(x, h)$. Then the iteration (1) has the form

$$x(t+h) = f(x(t), h) \quad (t = 0, h, \dots) \quad (12)$$

Suppose that $x(t)$ tends to a constant vector $\hat{x}$ as $t \rightarrow \infty$. Then from (2)–(4) we see that $p_i(x(t), h)$ tends to $p_i(\hat{x}, h)$. If $\hat{x}_i > 0$, we also have $\log x_i(t) \rightarrow \log \hat{x}_i$ and therefore $f_i(x(t), h)$ tends to $f_i(\hat{x}, h)$. If $x_i(t) > 0$ and $x_i(t)$ tends to $0 = \hat{x}_i$, then $p_i(x(t), h)$ tends to a positive constant and $f_i(x(t), h)$ tends to zero. Therefore, from (11), (12) we deduce that

$$\hat{x} = f(\hat{x}, h) \quad (13)$$

in either case. That is, $\hat{x}$ is a *fixed point* of the iteration defined by (12). The following theorem shows how to find all such fixed points.

Theorem 1. Consider the iteration defined by (1)–(4):

(a) if $x(t)$ has all components $x_i(t) \geq 0$ and if $x(t)$ tends to a constant vector $\hat{x}$ as $t \rightarrow \infty$, then (13) is satisfied.

(b) If $\hat{x}$ has all components $\hat{x}_i \geq 0$ and if $\hat{x}$ satisfies (13), then

$$\sum_{j=1}^m a_{ij} \hat{x}_j = 0 \quad \text{for all } i \text{ for which } \hat{x}_i \neq 0 \text{ or } 1 \quad (14)$$

Conversely, if $\hat{x}$ is a vector for which $\hat{x} \geq 0$ and for which (14) holds for every i for which $\hat{x}_i$ is neither 0 nor 1, then $\hat{x}$ satisfies (13).

Proof. Part (a) has already been proved and we turn to part (b). Assume that $\hat{x}_i \geq 0$ and $\hat{x}$ satisfies (13). If $\hat{x}_i \neq 0$, $\hat{x}_i \neq 1$, then from (13) and the definition of f , we deduce $p(\hat{x}, h) = 1$. Hence $q = r_i$ and

$$\sum_{k=1}^m a_{ik}^- \hat{x}_k = \sum_{k=1}^m a_{ik}^+ \hat{x}_k \quad (15)$$

From this equation, (14) follows. Conversely, let $\hat{x}$ be a vector with $\hat{x} \geq 0$ for which (14) holds. For any i for which $\hat{x} \neq 0$ or 1, we have (15). Therefore, $p_i(\hat{x}, h) = 1$ and $f_i(\hat{x}, h) = \hat{x}_i$. For any i for which $\hat{x}_i = 0$ or 1, we have $f_i(\hat{x}, h) = 0$ or 1 by definition, and again $f_i(\hat{x}, h) = \hat{x}_i$. Thus, (13) holds. This completes the proof.

An important conclusion to be drawn from Theorem 1 is that every equilibrium state (fixed point) of the iteration must have all its components equal to 0 or 1, unless the matrix A satisfies certain linear relations (14). In particular, if $\hat{x}$ is a fixed point with $0 < \hat{x}_i < 1$ for every i , then (14) must be satisfied for every i . This is equivalent to $A\hat{x} = 0$. Such an $\hat{x}$ therefore exists if and only if A is a singular matrix. This proves the following corollary.

Corollary. The iteration has a fixed point $\hat{x}$ with $0 < \hat{x}_i < 1$ for every i if and only if $\det A = 0$.

In general, we expect that the matrix A will be nonsingular, since the set of singular matrices is of 'low dimensionality' or 'probability' among the set of all matrices. Consequently, we can interpret the result as stating that the only equilibrium states $\hat{x}$ possible for the model are those in which at least one component $\hat{x}_i$ equals 0 or 1. That is, some subsystem variable must tend to its minimum or maximum allowable value. The fact that it is impossible to have an equilibrium of the system with all limits $\hat{x}_i$ strictly between 0 and 1 (unless A is singular) seems to indicate that these equations, in their present form, are inappropriate for models in which equilibrium state are expected. In the following section suggestions are made for

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extensions that may be more useful in simulation of behaviour over a long time span.

It is possible to analyse the local stability (local attractivity) of the fixed points. However, we omit this discussion here.

We shall now describe a numerical experiment which seems to reveal the presence of periodic behaviour of the iteration. For this purpose it was thought best to simplify by reducing the number of variables. Initially a system was formulated with only three variables (conceived as population, subsistence, and craft technology) controlled by the matrix

$$D = \begin{bmatrix} 0 & 0.5 & -0.9 \\ 0.4 & 0 & 0.2 \\ 0.5 & -0.1 & 0 \end{bmatrix}$$

Figure 5 shows two runs, each made with a time step $h=0.2$, and with respective initial values (a) 0.23, 0.95, 0.52; and (b) 0.1, 0.1, 0.1. For this matrix there is an equilibrium state with $\hat{x}_1 = 2/5$, $\hat{x}_2 = 1$, $\hat{x}_3 = 5/9$. In both runs, $x_2(t)$ quickly tends to 1, and $x_1(t)$ and $x_3(t)$ oscillate around the values $2/5$ and $5/9$, with apparently increasing amplitude.

Figure 6 shows runs made with the slightly altered matrix

$$E = \begin{bmatrix} -0.1 & 0.5 & -0.9 \\ 0.4 & 0 & 0.2 \\ 0.50 & -0.10 & -0.01 \end{bmatrix}$$

There is an equilibrium state $\hat{x}_1 = 1900/9002$, $\hat{x}_2 = 1$, $\hat{x}_3 = 4980/9002$. The runs were made with respective initial values (a) 0.05 + 1900/9002, 1, 4980/9002; (b) 0.01, 0.5, 0.01; and (c) 0.98, 0.98, 0.05. In all three runs, $x_2(t)$ tends to 1. In (b), $x_1(t)$ and $x_3(t)$ appear to approach periodicity, and in (c) they seem to fall to zero. In the event, fairly complex oscillatory behaviour seems to be possible for iterations of the form under consideration, presumably due to the nonlinearity of the governing equations.

Extensions and Alterations of the Model

A number of extensions of the model may be considered.

Interaction Coefficients Varying with Value of Variable

As described earlier each entry a_{ij} in the matrix A is fixed once and for all during an experiment. Although this assumption may be reasonable during the first simple stage of model building, it is clearly inappropriate in any real case in which long-term behaviour

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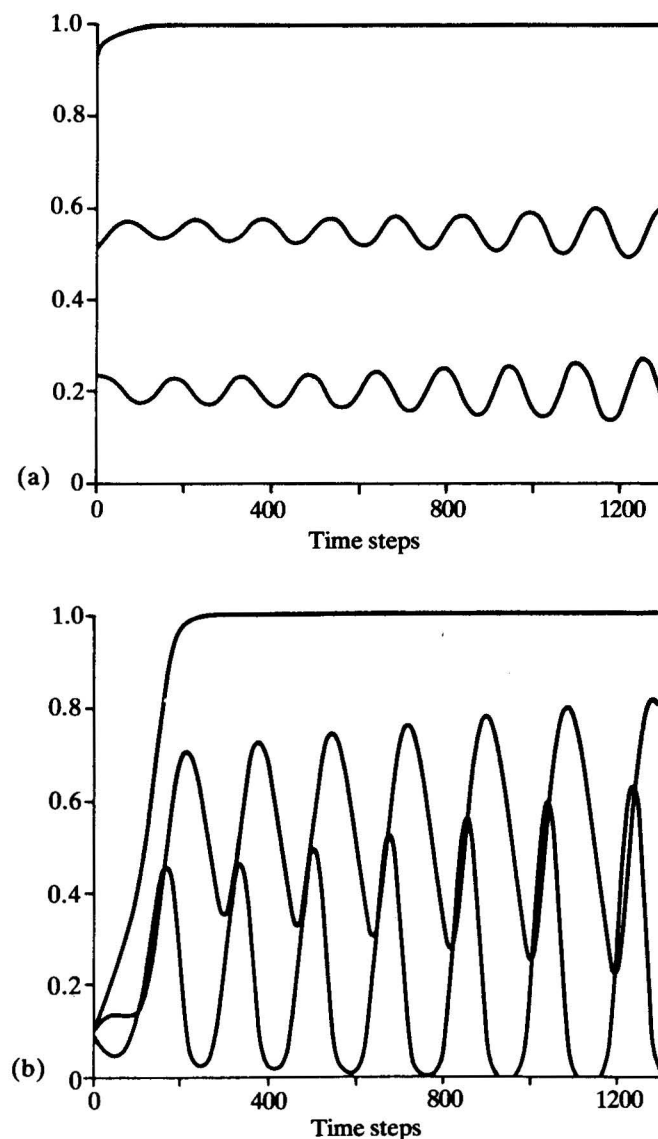


Figure 5. Growth pattern of the variables, using three variables with interactions indicated in matrix D and initial values (a) 0.23, 0.95, 0.52 and (b) 0.1, 0.1, 0.1.

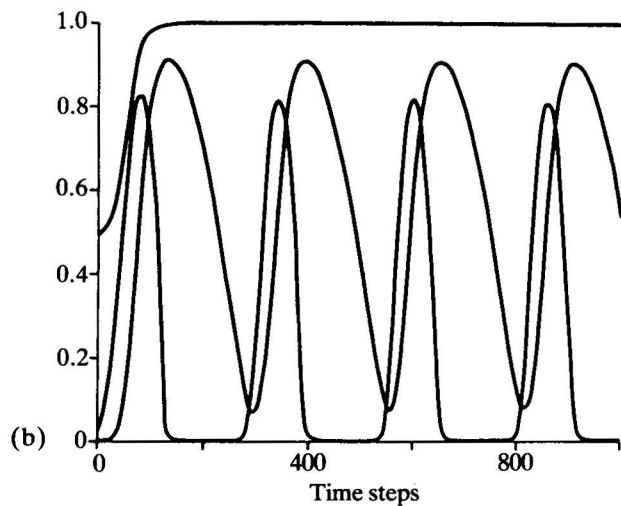
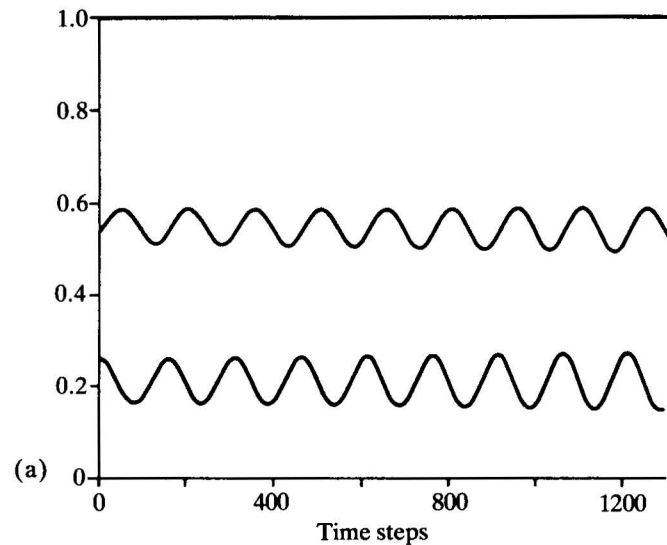
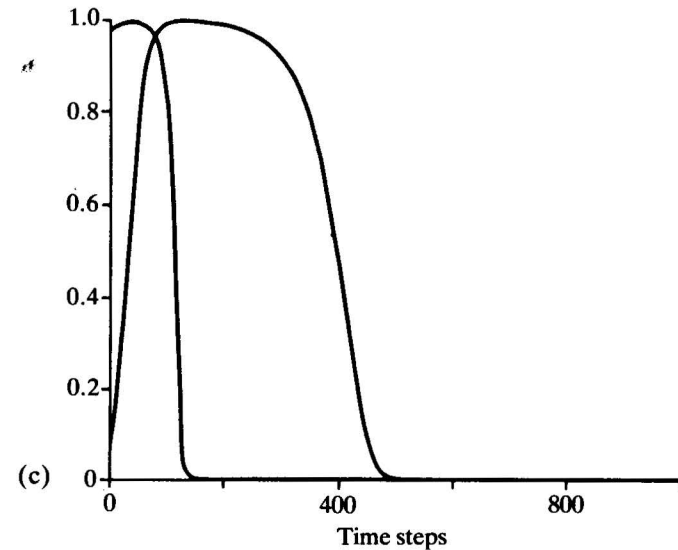


Figure 6. Growth behaviour pattern of the variables, with interactions indicated in matrix E and with initial values (a) 0.05 + 1900/9002, 1, 4980/9002 (b) 0.01, 0.5, 0.01 (c) 0.98, 0.98, 0.05.



is considered. It may often be the case that an influence is present for small values of a variable but absent for larger values. In some cases the influence could become negative for large values, suggesting some kind of negative feedback. In general, then, the numbers a_{ij} should be allowed to be *state dependent* (i.e., functions of the values of some or all of the variables). In other words, the linear forms in equations (7) and (8) should be replaced by

$$q_i = 1 + h \sum_{k=1}^m a_{ik}^-(x)$$

$$r_i = 1 + h \sum_{k=1}^m a_{ik}^+(x)$$

where $a_{ik}(x) \equiv a_{ik}(x_1, x_2, \dots, x_m)$ is an arbitrary function of the variables $x_1, x_2, \dots, x_m$, and where $a_{ik}^+(x) = a_{ik}(x)$ for $a_{ik}(x) > 0$ and $a_{ik}^-(x) = |a_{ik}(x)|$ for $a_{ik}(x) < 0$. In practice, one might begin by taking $a_{ik}(x)$ to be a quadratic function of x_k only, for example,

$$a_{ik} = (1 - c_{ik}x_k)x_k$$

No investigation of such models has been undertaken so far by us, although one can readily visualise that, with suitably formulated relationships, the stability of the system could be attained.

Interaction Coefficients Varying with Rate of Change of Variables

The difficulties in achieving a steady rate, or a number of steady states, with a linear dependence of q_i and r_i upon the value of the variable suggested the idea that in some cases they may depend rather on whether the variable in question is increasing or decreasing. For instance, if we are considering the influence of population on subsistence production, a stable, nonchanging population might be thought to induce no change in the subsistence system. Indeed, when we are trying to set up a homeostatic system, some formulation of the following form may be appropriate:

$$q_i = 1 + \frac{1}{2}h \sum_{k=1}^m \left\{ \left| a_{ik} + \frac{b_{ik}}{x_k} \frac{dx_k}{dt} \right| - \left(a_{ik} + \frac{b_{ik}}{x_k} \frac{dx_k}{dt} \right) \right\} x_k$$

$$r_i = 1 + \frac{1}{2}h \sum_{k=1}^m \left\{ \left| a_{ik} + \frac{b_{ik}}{x_k} \frac{dx_k}{dt} \right| + \left(a_{ik} + \frac{b_{ik}}{x_k} \frac{dx_k}{dt} \right) \right\} x_k$$

Equations of this kind have been suggested by Kane, Thompson, and Vertinsky. We have not yet investigated such models. In a discrete time simulation, the derivatives dx_k/dt can be approximated by finite differences. A listing of a large FORTRAN program for systems of this type has been published by Thompson, Vertinsky and Kane (1974).

Time Lags. In many systems, changes in the present are determined not by the current values of variables but by their values some time in the past. This can easily be built into the model by allowing q_i and r_i to depend not only on $x_k(t)$ but also on x_k a given number of time steps earlier.

Alternative Forms of Equation (1). Although the iteration equations (1)–(4) have certain convenient features, a number of other formulations in terms of differential and difference equations are possible. Some of these may be more useful by allowing 'physical' interpretation of the parameters and by allowing the insertion of equations which reflect the supposed interactions more realistically and in more detail. A number of different mathematical formulations should be tried and compared.

Sensitivity Analysis. In evaluating a model such as the one described here, it is usually important to conduct a sensitivity analysis. That is, one endeavours to see how great the variation is in various performance measures when the parameters of the model are varied by a designated amount. We have not carried out such an analysis for the

present model since we feel that the whole structure of the model is still under investigation.

Further Outlook

In the present early stage of development, the model proposed cannot be regarded as simulating in any meaningful sense the early development of Aegean civilisation. That specific cultural context has simply been used as the starting point for the formulation of a much more general model, and the exploration of some of its properties.

Indeed, the foregoing discussion allows us to consider more clearly what sort of behaviour would be required of such a model in order that we should feel its behaviour modelled satisfactorily some aspects of the behaviour of the real world. At least four may be suggested:

1. The rate of change and behaviour of the variables should be consonant with that of the limits in the range of real behaviour which they represent.
2. The model should display homeostatic behaviour, in the sense not of absolutely stable states, but of periods of very gradual change.
3. The model should display sustained growth under certain circumstances.
4. Ideally it should also display threshold effects – that is, periods of sustained and fairly rapid growth between periods of relative stability.

We suggest that this approach brings out in an informative way a number of the difficulties inherent in models of system dynamics. In particular it suggests that lengthy discussions of input (or output) data can usefully be deferred until the range of behaviour of the model, when subjected to a wide variety of input data, has been examined and found plausible. Already this approach has led us to consider more closely the mechanism of homeostasis within the culture system. And already it has led us to feel more acutely the danger of ascribing to the real world aspects of model behaviour which instead arise from the constraints built into the model. We were particularly struck by the difficulty in simulating homeostatic behaviour with this model when the values of the coefficients were constant. Indeed it occurred to us to wonder whether this is not a limitation which applies also, although for different reasons, to the model, based on the work of Forrester, used for the study *The Limits of Growth* (Meadows *et al.* 1972). It is apparently the case that not one of the variables studied there in a number of different

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simulations reaches a steady state within the time span considered, until the population is artificially and externally held constant (ibid. 160).

The aim of the present work is indeed to arrive at (approximately) steady states, but these should arise from the interactions of the variables rather than be imposed quite arbitrarily from the outside.

[1979]

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12

THE SIMULATOR AS DEMIURGE

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Demiurge: (Greek *demios*, for the people, *ergos*, work): 1a, a Platonic subordinate deity who fashions the sensible world in the light of eternal ideas; 1b, a Gnostic subordinate deity who is the creator of the material world; 2, something that is an autonomous creative force or decisive power.

Chambers Twentieth Century Dictionary

Systems thinking has increasingly been regarded as an essential component of archaeological theory since its introduction to the subject nearly two decades ago (Binford 1962). Yet despite the undoubted attractions of simulating the past, usually within some appropriate systems framework, simulations in this field have so far been rather few in number (Thomas 1972, Cordell 1975 and Zubrow 1975 being among the first). Only very recently has system dynamics modelling been applied to past cultural systems (Hosler, Sabloff and Runge 1977; Shantzis and Behrens 1973).

It is the very essence of simulation modelling that the simulator creates a device which will produce some representation of reality. A good simulation mimics reality closely. But a successful simulation is rarely achieved at the first attempt. If at first you don't succeed you try again, modifying or tuning the device to give a better - that is, closer to real - output. The simulator proceeds in some senses very much like an inventor, trying to get his infernal device to work. The theoretical content of his creation is often not altogether clear, being sometimes implicit in the structure of his invention, rather than set out in advance, neatly and explicitly, in the form of a number of theoretical relationships.

These two aspects - the frequent necessity for successive attempts before it 'works,' and the deliberate anticipation of 'unexpected' behaviour - often combine to make systems modelling seem a very